Simultaneous Computation of Heat Transfer and Dispersion Coefficients and Thermal Conductivity Value in a Packed Bed of Spheres:

II. Technique of Computing Numerical Values

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The method of computing the numerical values is given. It is illustrated with a simulated example to show the high accuracy that is possible, the unknowns being found to four significant figures. The only quantities required are ones that can be measured accurately: the radius of the spherical packing, its volumetric specific heat, and values of the amplitude attenuation and phase angle lag at a minimum of four finite values of the frequency.

The procedure for computation is outlined; it is an improvement over that suggested in Part I (*1*) in that the difficult task of finding t_{lag} at zero frequency is avoided. In its place there is the requirement only that the response of the system be determined at a minimum of four frequencies.

ARGUMENT

For a set (designated by p) of any three frequencies the line $(f_1)_p = 0$, $p = 1$ in the h, k plane will contain not only the value of h_i and the value of k_i that simultaneously correspond to the true (but unknown) value of Z, but also an infinite number of other solutions that correspond to other values of Z. For any other set of values of frequency, differing from the first, a new line $(f_1)_p = 0$, $p = 2$ can be generated that also will contain h_i and k_i together with another infinite number of combinations of values h and k, each combination corresponding to a certain value of Z. Thus a point of intersection of $(f_1)_{p=1} = 0$ and $(f_1)_{p=2} = 0$ will be the true value of h_i, k_i and this is thus a necessary condition; it may not be *sufficient* to find h_1 and k_1 where $i = 1$ denotes the true value, because the lines $(f_1)_p = 0$, $p = 1, 2$ may also intersect at other points h_i k_i in the h, k plane, where $i = 2, 3 \ldots$ denotes spurious intersection values of h and k. If therefore a third, fourth, ... set of frequencies is chosen to give additional

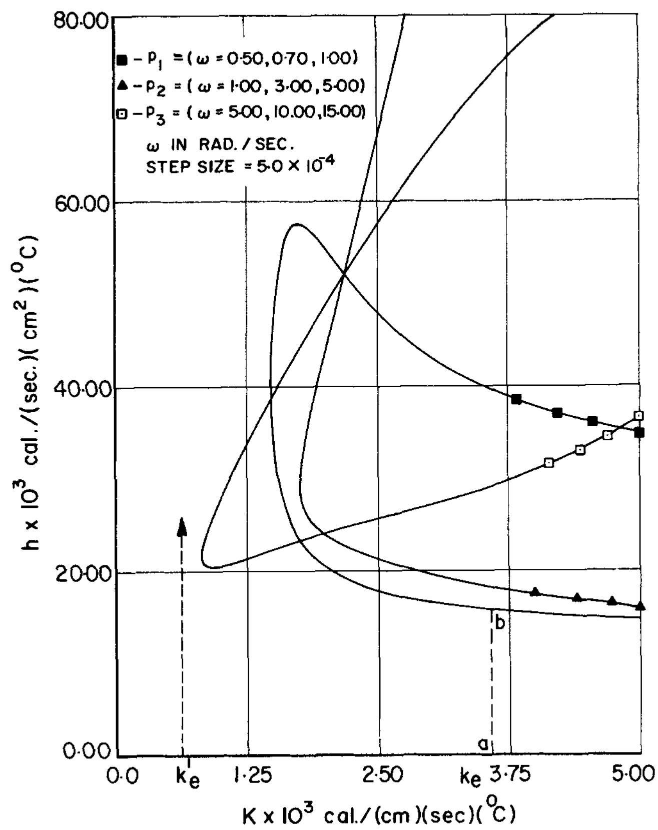

Fig. 1. Illustrating the method: the search for the $(f_1)_p = 0$ curves in the $(k, h, 0)$ plane.

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lines $(f_1)_{p=3} = 0$, $(f_1)_{p=4} = 0$, ..., they must all intersect at h_1, k_1 but will not in general have any other common intersection point. This is illustrated in Figure 1. Hence if response measurements are made at frequencies ω_j sufficient to obtain at least three surfaces F_p $(f_1, h, k) = 0$, then if Equation (6) is written as

$$f_{1_p}(h, k) \equiv K_1(\lambda_1/\lambda_3) + K_2(\lambda_2/\lambda_3) + 1 = 0 \quad (6a)$$

the lines $f_{1_p} = 0$; $p = 1, 2, 3, \ldots$ are where the surfaces intersect the $(0, h, k)$ plane. Since a minimum of three surfaces are required, then $j \geqq 4$. To be distinct, only one of the frequencies needs to be different between sets (within a set, $j \neq j$). Hence in general the number of distinct surfaces is $p = {}_jC_3$. The common intersection of the three (or more) lines where the surfaces intersect the $(0, h, k)$ plane gives the desired results, h_1 and k_1.

PROCEDURE

Figure 1 contains some typical shapes of the curves $f_{1_p} = 0$. These are of course not known, so a search procedure is performed. An initial estimated value (k_e) of k is used and values of f_{1_p} $(h, k)_{p=1}$ are calculated, with h gradually increasing from zero in predetermined steps, thus moving along the path a b in the $(0, h, k)$ plane. The calculation is continued until a value of h is obtained that causes f_{11} (h, k) to intersect the $(h, k, 0)$ plane at point b on the line $f_{11} = 0$. Then from point b the curve f_{11} $(h, k) = 0$ is generated by using a local search routine by incremental changes in the values of h and k, the step sizes being specified as input information. Furthermore the choice of k_e is circumscribed. For example a choice such as k'_e would not intersect the $f_{11} = 0$ line. The procedure is then repeated for all the p surfaces, and the common intersection of the f_{1_p} $(h, k) = 0$ lines gives the desired values of h_i and k_i. Now substitution for Z from Equation

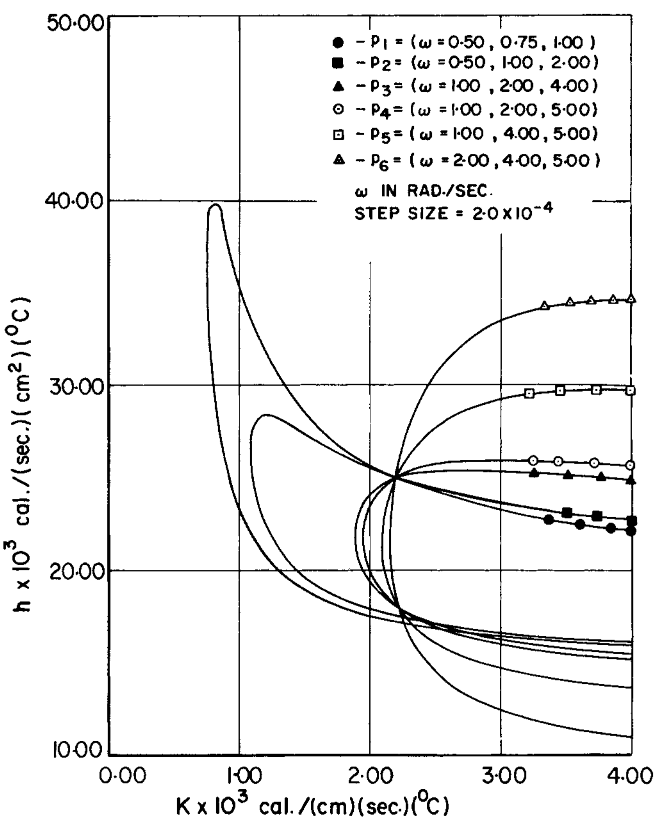

Fig. 2. Six $(f_1)_p = 0$ curves showing true and spurious intersections in the numerical example.

TABLE 1. VALUES IN THE SIMULATION

(a) Simulated Experimental Values

$r_0 = 0.10$ cm. $\quad (\rho c)_s = 0.5$ cal./(cc.)(°C.)

j	ω_j rad./sec.	Π	ψ
1	0.500	−0.7469276	−1.555774
2	0.750	−1.348552	−1.892263
3	1.000	−1.885410	−2.005146
4	1.600	−2.760541	−1.881237
5	2.000	−3.100429	−1.720186
6	3.000	−3.545721	−1.378405
7	4.000	−3.751077	−1.157595
8	5.000	−3.870130	−1.014479

(b) Additional Values Needed for the Simulation

$(\rho c)_g = 3.1 \times 10^{-4}$ cal./(cc.)(°C.)
$\epsilon = 0.295$
$U = 2500$ cm./sec.
$L = 2.50$ cm.
$D = 250$ sq.cm./sec.
$h = 25 \times 10^{-3}$ cal./(sec.)(sq.cm.)(°C.)
$k = 2.2 \times 10^{-3}$ cal./(sec.)(cm.)(°C.)
Hence $N_{Pe} = 25.00$ [and so $(N_{Pe})_{\text{particle}} = 2.00$]

(7), which must hold at h_1 and k_1, and subsequent rearrangement of Equation (5) gives

$$N_{Pe} = \nu_2 \Bigg/ \left[\left(\frac{\Pi_3\nu_2 - \Pi_2\nu_3}{\nu_3\lambda_2 - \nu_2\lambda_3} \right) \lambda_2 + \Pi_2 \right] \quad (8)^*$$

where the numerical subscripts represent one of three possible choices of the three frequencies which define a surface p, that is, any one of Equations (5a), (5b), and (5c) can be used. All the quantities on the right-hand side of Equation (8) are known and so N_{Pe} may be calculated.

The second, third, ... intersections of the $f_{1_p} = 0$ lines are different from each other only if the span of ω_j is sufficiently large. If not, and two roots are obtained, then the value of N_{Pe} for both roots must be computed; only one of them will give a positive value of N_{Pe}.

Sets of frequencies should be chosen such that the lines intersect at as large an angle as possible; the upper limit will probably be set by experimental difficulties, the lower one by the condition that the value of λ must be larger than about 0.15.

COMPUTER PROGRAMS

The simulation programs of Part I have been rewritten for an IBM 360-75 computer. In addition new programs have been written to obtain solutions by the method outlined in this paper; the listings are given in reference 2. The results are plotted automatically to give the common intersection points (h_i, k_i).

After the approximate position of the intersections has been found, increased accuracy may be obtained by enlarging the scale on the graph or (a method that reduces the computational time to seconds) a Newton-Raphson search is used.

For an experimental determination of h, k, and N_{Pe}, it is necessary to know only the following quantities: radius of the spherical packing r_0, the volumetric specific

* Equations are numbered consecutively from Part 1.

TABLE 2. FINAL NUMERICAL RESULTS

Variable	Input value	Newton-Raphson value
h	25.000×10^{-3}	25.00×10^{-3}
k	2.2000×10^{-3}	2.200×10^{-3}
N_{Pe}	25.000	25.00
$(N_{Pe})_{\text{particle}}$	2.0000	2.000
$\lambda_{(\omega = 0.75)}$	0.221049	0.2210
$\lambda_{(\omega = 2.00)}$	0.582266	0.5823
$\lambda_{(\omega = 4.00)}$	0.736841	0.7368
$\lambda_{(\omega = 5.00)}$	0.765859	0.7659
f_{1p}	0.00000	

heat of the spherical packing $(\rho c)_s$, the values of $\Pi = \ln (A_{\text{out}}/A_{\text{in}})$, and the phase lag ψ at j values of the frequency.

(SIMULATED) EXAMPLE

The values of the above quantities are given in Table 1; those of Π and ψ were computed for a number of frequencies and the underlined values were those used in the numerical computation.

Figure 2 shows the curves $f_{1p}(h, k) = 0$ for the various combinations of ω_j ($j = 1 \ldots 6$). The true root was found by calculating N_{Pe} from Equation (8) and eliminating those that gave negative values. Table 2 lists the final values obtained; it will be seen that the unknowns h, k, and N_{Pe} were recaptured to four significant figures by the Newton-Raphson method. (At least one graph is recommended to determine the search area and because visualization of the shape of the $f_{1p} = 0$ lines helps in planning the computational attack.)

ACKNOWLEDGMENT

This work was carried out with financial support from the National Research Council of Canada; our thanks are due to this body for the operating grants and for the award of a postgraduate scholarship to M. J. Goss.

The authors are most grateful to V. Taht of the Statistics Department, University of Waterloo, Waterloo, Ontario, for writing and converting programs.

NOTATION

k_e, k'_e = initial trial estimate of k

j = number of discrete values of the frequency

N_{Pe} = UL/D = tube Peclet number, dimensionless

p = p^{th} set of three frequencies

r_0 = radius of the spherical packing, cm.

Subscript

$i(= 1, 2 \ldots)$ = i^{th} root satisfying $(f_1)_p = 0$, p = two values

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Manuscript received August 5, 1969; revision received April 15, 1970; paper accepted April 23, 1970.

III. Experimental Method and Results

This part describes an experimental essay at finding simultaneously the values of the heat transfer coefficient, the dispersion coefficient, and the thermal conductivity of the packing in a packed bed of spheres. The logic of the method was presented in Part I, while the computational technique of calculating the numerical values of these quantities was presented in Part II. A sufficient number of experimental results are presented to demonstrate the potential of the method as a means of obtaining information that has previously been obtainable either with great uncertainty or not at all.

A method of finding three unknown parameters—heat transfer coefficient, thermal conductivity of the packing, and the longitudinal Peclet number—from a dispersion model of a packed bed of spheres with fluid flowing through it has been described in Part I (*1*) and demonstrated in Part II (*2*) by simulation. It was shown that the values of these parameters could be obtained to four significant figures with modest amounts of computer time, but that the allowable errors in the input quantities had to be small. This seemed likely to present an experimental challenge, and this part describes the present state of an apparatus which has produced some results. Three of these are presented: two which show that the measured values were close to reported values, while the third, although producing results of the right order of magnitude, is put in to demonstrate why care is needed in choosing ranges of the input quantities.

APPARATUS

General

To use the method described in Parts I and II, the physical system had to be made to agree with the model as closely as possible. The aim was for tolerances of meaningful quantities not to exceed one part in a thousand. Thus the temperature wave was to be of constant amplitude, of constant known frequency, and was to have as low and constant a harmonic content as possible in order to facilitate measurement.

The fluid flow was to be constant, nonrotational, and parallel to the axis of the system. Radial variations of velocity and temperature were to be small (within a few percent of the mean) at least in the central measurement region.

The apparatus is shown in Figure 1; further details are given in reference 9.

Air, supplied by the compressor 2, was cooled to its dew point in order that subsequent heating by the temperature controller raised the mean temperature to that of the laboratory. The interchanger used a glycol solution cooled in tank 11 by refrigeration unit 9. The cooled air passed downward through a vacuum jacketed glass tube shown in Figure 2 (8.6 cm. I.D. by 186 cm. long) containing the temperature control devices, the bed with its associated thermometers, and the flow straighteners. The air, after emerging from the end of this vacuum jacketed tube (which was in addition wrapped in 1 in. styrofoam insulation), passed upward, outside it, in an annular space inside tube 20. By this procedure it was possible to enjoy a combination of effective insulation and small temperature gradients. A rough calculation gave for the worst case (namely,